Mathematical Association of America
Wisconsin Section
Mathematics Contest Examination
December 1, 2011

1. Do not open this booklet until you are directed to do so.

2. This is a multiple choice test. Each multiple choice question has five possible answers, exactly one of which is correct. You are to circle the letter corresponding to the correct response on the answer sheet for as many problems as you can do in the 75 minutes allowed.

   EXAMPLE:
   If x is 3 and y is 4 then $2x - y$ is

   (a) –1 (b) 0 (c) 1 (d) 2 (e) none of these.

3. Use pencil or pen. A sheet of paper will be provided for your scratch work. Calculators may be used. Tables, books, notes, etc. may not be used.

4. The scoring system has been set up to give more credit in the long run for leaving a question unanswered than for guessing rashly. On the other hand, whenever you can eliminate three possibilities, it is better to guess between the remaining two possibilities than to leave the question unanswered.

5. Fill in the following blank and wait for the signal to start the examination.

PRINT____
First Name Last Name

Your teacher will fill in the following blanks:

Part	Number of Questions	Number Right	Number Not Answered
1	8	____x 4 =____	____x 1 = ____
2	8	____x 8 = ____	____x 2 =____
3	2	____x 12 = ____	____x 3 = ____
Total	18	Sub-Total _____	Sub-Total ____

Score (Sum of both sub-totals)_____

Part I:

1. Suppose TP is the diameter of a circle and that Q is a point on the circle with the arc distance from Q to T equal to 1/10 of the circumference of the circle. Determine the measure of angle TPQ in radians.

(a) $\pi/6$ (b) $\pi/10$ (c) $\pi/12$ (d) $\pi/14$ (e) $\pi/15$

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2. Among the three inequalities below, find all whose solution set is the same as the solution set for $(x+2)(x-3)\le 0$.

(1) $(x+2)^3(x-3)\le 0$ (2) $\frac{x-3}{x+2}\le 0$ (3) $\frac{(x+2)(x-3)}{x^2}\le 0$

(a) (1) (b) (2) (c) (3) (d) (1) and (2) (e) (2) and (3)

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3. If $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$, then what is $\frac{x-y-z}{x+y+z}$?

(a) -5/6 (b) -5/9 (c) 5/9 (d) 5/8 (e) 5/6

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4. When $0<a<1$, simplify the expression $\sqrt{\left(a-\frac{1}{a}\right)^2+2}-\left|a+\frac{1}{a}\right|$.

(a) $-2a+2$ (b) $2a+2$ (c) $\frac{2}{a}+2$ (d) $2a-\frac{2}{a}$ (e) 2

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5. The product of 2011 integers is 1. Which of the following **CANNOT** be the sum of these integers?

(a) 2011 (b) 2007 (c) 2003 (d) 2001 (e) 1999

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6. The value of the product $\log_2 3\log_3 4\log_4 5\cdots\log_{2^n-1}2^n$ is

(a) $\log_{2^n}(2^n+1)$ (b) $\log_{2^n-1}2^n$ (c) 2^{n-1} (d) 2^n (e) n

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7. Given that $\begin{array}{l} A+B=2011 \\ C+D=2011 \\ A-D=2011 \end{array}$, then $A-B+C-D=$

(a) 2(C-D) (b) 2(C+D) (c) 2(D-C) (d) 2(A-B) (e) 2(B-A)

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8. To celebrate her birthday earlier this year on May 7th (05/07/2011), a math teacher had her class calculate 7^{2011} and divide the answer by 5. What was the correct remainder after this division was performed?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

Part II:

9. At a certain high school 20% of the upperclassmen wear glasses and 20% of the students who wear glasses are upperclassmen. If 35% of the underclassmen wear glasses what fraction of the student body wear glasses?

(a) 3/10 (b) 4/13 (c) 7/23 (d) 9/29 (e) 10/33

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10. If x and y are complex numbers with $x + y = x^3 + y^3 = 4$, what is the value of $x^6 + y^6$?

(a) -123 (b) -234 (c) -345 (d) -456 (e) none of these

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11. A fair six-sided number cube (i.e., a die) is tossed. If the result is odd then the die is tossed once more. If the original result is even, then the die is tossed twice more. What is the probability that the sum of the results is seven?

(a) 1/7 (b) 1/9 (c) 1/12 (d) 5/36 (e) 31/216

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12. For real numbers x, y, and z,

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2 \qquad \text{and} \qquad 2^x=3^y=5^z=k.$$

What is k?

(a) $\sqrt{20}$ (b) $\sqrt{30}$ (c) $\sqrt{35}$ (d) $\sqrt{60}$ (e) $\sqrt{90}$

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13. Let x be a four digit positive integer such that $y = 4x$ is equal to the integer obtained by reversing the digits of x. Which of the following statements must be true?

I. The thousands digit of the number is 2
II. The hundreds digit of the number is 1
III. There can only be one such number

(a) I only (b) I and II only (c) II and III only (d) I and III only (e) All three are true

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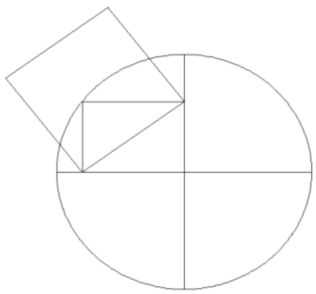
14. A rectangle is constructed in the second quadrant of a circle, as shown, so that its height is 60% of the radius. A square is then constructed whose base is the diagonal of the rectangle. The area of the rectangle is what percentage of the area of the square?



(a) 46% (b) 48% (c) 50% (d) 52% (e) 54%

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15. For any polyhedron, the number of Faces (F), the number of Edges (E) and the number of Vertices (V) obey the formula F+V=E+2. A polyhedron with every face a triangle (each face has three neighbors, and triangles touching only at corners do not count as neighbours) has 2011 vertices. How many faces does it have?

(a) It is impossible for such a polyhedron to exist
(b) Not enough information given to solve the problem
(c) 4018
(d) 4019
(e) 4020

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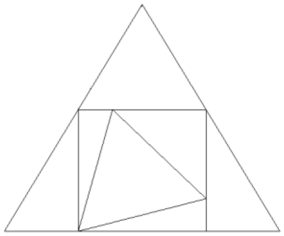
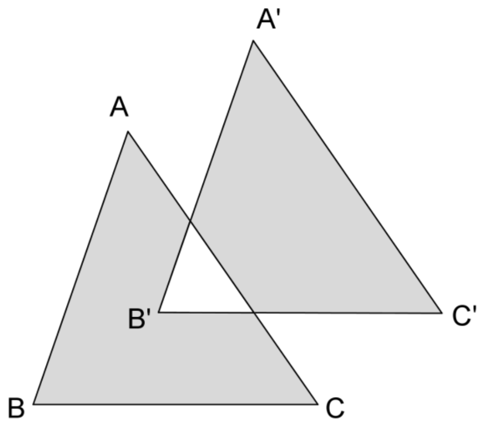
16. An equilateral triangle is placed inside of a square, which is placed inside of another equilateral triangle (see diagram). If the length of each side of the larger triangle is one meter, then what is the area(in square meters) of the smaller triangle?



a) $2\sqrt{3} - 3$ b) $(2\sqrt{3} - 3)^2$ c) $(2\sqrt{3} - 3)^3$ d) $(2\sqrt{3} - 3)^4$ e) $(2\sqrt{3} - 3)^5$

Part III:

17. The area of ΔABC is $27cm^2$. Move ΔABC and name it ΔA'B'C'. If all sides of ΔA'B'C' are parallel to the original sides of ΔABC, and if B' is at the center of mass of ΔABC (otherwise known as the centroid or the intersection of the three medians), as shown in the figure below, find the area of the shaded region.



(a)52 (b)51 (c)50 (d)49 (e)48

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18) A Unit Fraction or Reciprocal is defined as a fraction where the numerator is 1 and the denominator is a natural number.

Consider the sequence of rational numbers between 0 and 1, $\left\{\frac{1}{2},\frac{1}{3},\frac{2}{3},\frac{1}{4},\frac{2}{4},\frac{3}{4},\frac{1}{5},\cdots\right\}$. We wish to write each number in the list as either a single reciprocal, $\left(\frac{1}{a}\right)$, or as the sum of a collection of **different** reciprocals ,$\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\cdots\right)$, (so 2/3=1/3+1/3 doesn't count, but 2/3=1/2+1/6 does). What is the first rational number in the sequence that needs more than two different reciprocals to express it as such a sum?

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{2}{7}$ (e) $\frac{3}{7}$
