Mathematical Association of America
Wisconsin Section
Mathematics Contest Examination
December 6, 2012

1. Do not open this booklet until you are directed to do so.

2. This is a multiple choice test. Each multiple choice question has five possible answers, exactly one of which is correct. You are to circle the letter corresponding to the correct response on the answer sheet for as many problems as you can do in the 75 minutes allowed.

   EXAMPLE:
   If x is 3 and y is 4 then $2x - y$ is

   (a) –1   (b) 0   (c) 1   (d) 2   (e) none of these.

3. Use pencil or pen. A sheet of paper will be provided for your scratch work. Calculators may be used. Tables, books, notes, etc. may not be used.

4. The scoring system has been set up to give more credit in the long run for leaving a question unanswered than for guessing rashly. Notice than an unanswered question earns points but that an incorrectly answered question earns none. On the other hand, whenever you can eliminate three possibilities, it is better to guess between the remaining two possibilities than to leave the question unanswered.

5. Fill in the following blank and wait for the signal to start the examination.

PRINT_____
        First Name   Last Name

Your teacher will fill in the following blanks:

Part	Number of Questions	Number Right	Number Not Answered
1	8	____x 4 =____	____x 1 = ____
2	8	____x 8 = ____	____x 2 =____
3	2	____x 12 = ____	____x 3 = ____
Total	18	Sub-Total _____	Sub-Total ____

Score (Sum of both sub-totals)_____

1. Three motorcyclists ride around a circular race course starting at the same place and the same the same time. The first passes the starting point every 12 minutes, the second every 18 minutes and the third every 16 minutes. When will all three pass the starting point again at the same time?

    (a) 2hrs   (b) 2hrs 6mins   (c) 2hrs 12mins   (d) 2hrs 18 mins   (e) 2hrs 24mins

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2. A student rolled a fair die until the sum of her scores was a prime number. She had to roll three times. What is the probability that her last throw was a 6?

(a) 0   (b) 1/24   (c) 2/24   (d) 6/24   (e) 7/24

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3. In the sequence of grouped numbers below, find the second number in the sixth group;

                (1), (2,4), (8, 16, 32), ….

(a) 2^{14} (b) 2^{15} (c) 2^{16} (d) 2^{17} (e) 2^{18}

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4. Intelligent life has been found in a planet named Zarbi. Zarbians actually use a base other than 10. They represent 1,2,3,4,5 by the following symbols.

Number	Symbol
1	⌠
2	⌐
3	◊⌠
4	⌠⌠
5	⌐⌠

How will Zarbians denote 6?

a) ◊⌐   (b) ⌐◊   (c) ⌒   (d) ◊⌠⌠   (e) ◊⌐⌐

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5. A swimming pool is 45 ft long and 30 ft wide. The bottom of the pool is an inclined plane which is 3 ft deep at one end and is 12 ft deep at the other end. If the pool currently holds 3675 cubic feet of water, what is the approximate maximum depth of water in the pool at the moment?

(a) 6 ft   (b) 7 ft   (c) 8 ft   (d) 9 ft   (e) 12 ft

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6. I have a watch that gains a minute every day and a clock that loses two minutes every day. Both time pieces give hours from 1 to 12 (neither is a 24 hour timepiece). I set them both to 12 at midnight. What time will they show when they both show the same time again?

(a) 2:00   (b) 4:00   (c) 6:00   (d) 8:00   (e) 12:00

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7. Find the sum of all the numbers in the following array:

1	2	3	4	...	100
2	4	6	8	...	200
3	6	9	12	...	300
.	.	.	.	...	.
.	.	.	.	...	.
.	.	.	.	...	.
100	200	300	400	...	10,000

(a) 25002500 (b) 25,022,500 (c) 25,502,500 (d) 25,002,550 (e) 22,502,500

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8. The inversion of a point P with respect to a circle centered at O and having radius r is the point Q on the line OP so that OP x OQ = r^2. In the figure below, there are two concentric circles centered at O with radii 4 and 8 units, and the lines CO and OD are perpendicular. Find the area of the region that is the inversion of the region ABCD with respect to the larger circle.

C
B
A
D
O

(a) 12π sq units   (b) 16π sq units   (c) 36π sq units   (d) 48π sq units   (e) 64π sq units
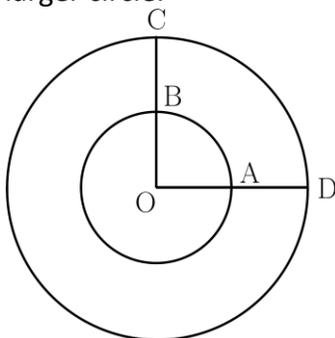
9. The diagram consists of 3 isosceles acute triangles and three isosceles obtuse triangles. The acute triangles are all congruent or similar to each other and the obtuse triangles are all congruent or similar to each other. In any of the triangles, what is the ratio of long side length to short side length?
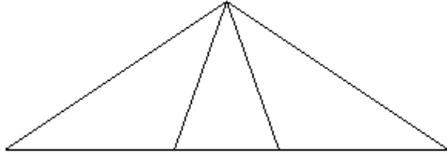

(a) $\sqrt{5} + 1:2$ (b) $\sqrt{5} - 1:1$ (c) $\sqrt{3} + 1:\sqrt{2}$ (d) $\sqrt{6} + 2:3$ (e) $\sqrt{6} - 2:2$

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10. If three fair, 7-sided dice, with faces numbered 2,3,4,...,8, are rolled at the same time, what is the probability that the largest result on one is strictly larger than the total of the other two?

(a) 57/343   (b) 58/343   (c) 59/342   (d) 60/343   (e) 61/343

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11. An <u>infinite</u> number of squares are to be inscribed in an equilateral triangle as in the diagram. What percentage of the triangle is covered by squares?
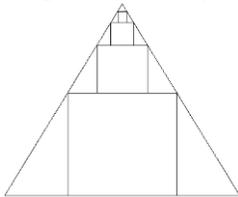
(a) 63.4   (b) 63.5   (c) 63.6   (d) 63.6   (e) 63.7

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12. Alicia, Bruno and Caleb were hosting a party and they each spent $9 on supplies. Everyone bought snacks, desserts and drinks, but, although everyone spent a whole dollar amount on each item, no one spent the same on any two items, nor was anyone's expenditure identical to that of anyone else. They discovered that nobody else spent as much on anything as Bruno did on deserts, but that Alicia spent twice as much on snacks as she did on deserts. Moreover, a total of $9 was spent on each item. How much did Caleb spend on drink?

(a) $1   (b) $2   (c) $3   (d) $4   (e) $5

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13. A 3 inch x 3 inch cube of wood is painted red on each of its six faces. It is then sliced into twenty seven 1x1 cubes, which are put into a bag. Two cubes are taken out at random. What is the probability that the difference in number of painted faces on these two cubes is an odd number?

(a)180/351   (b) 182/351   (c) 184/351   (d) 186/351   (e) 188/351

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14.   A circle is inscribed in a 3,4,5 triangle and another is circumscribed around the same triangle. What is the area of the region between the two circles?

(a) 5.25π   (b) 5.5π   (c) 5.75π   (d) 6π   (e) 6.25π

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15.   The approximate radius of the circle which is the intersection of
     $x + y + z = 1$ and $x^2 + y^2 + z^2 = 1$ is

        (a) 0.8163   (b) 0.8164   (c) 0.8165   (d) 0.8166   (e) 0.8167

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16. Two players(Black and White) are finishing a game as shown:

Black		White		Finish
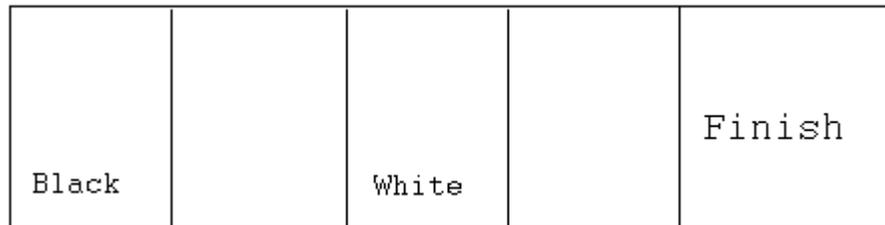
Each person in turn rolls a die. If someone lands on a square already occupied, that person takes an extra turn. To win one must reach or pass the finish square. It is Black's turn. What is the probability that Black wins?

a) 0.662   b) 0.663   c) 0.664   d) 0.665   e) 0.666

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17. Consider the equation TWO + TWO = FOUR where each letter is assigned a distinct nonzero digit(1,2,3,4,5,6,7,8,9). In how many ways can the assignments be made to get a true equation?

   (a) 5 (b) 6 (c) 7 (d) 8 (e) 9

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18) Five ants are placed along a 1 meter rod. Some face to the left and some to the right. One ant is placed exactly at the 50cm point, but the others are placed randomly. Each ant moves at a constant 1cm/sec, and when it reaches the end of the rod, it falls off. The instant two ants meet, they change directions. How long do we have to wait before we know that the ant that was placed at the 50cm point has fallen of the rod?

   (a) 50 sec   (b) 75 sec   (c) 87.5 sec   (d) 93.75 sec (e) 100 sec

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