Mathematical Association of America
Wisconsin Section
Mathematics Contest Examination
December 3, 2015

1. Do not open this booklet until you are directed to do so.

2. This is a multiple choice test. Each multiple choice question has five possible answers, exactly one of which is correct. You are to circle the letter corresponding to the correct response on the answer sheet for as many problems as you can do in the 75 minutes allowed.

   EXAMPLE:
   If x is 3 and y is 4 then $2x - y$ is

   (a) –1 (b) 0 (c) 1 ((d) 2) (e) none of these.

3. Use pencil or pen. A sheet of paper will be provided for your scratch work. Calculators may be used. Tables, books, notes, etc. may not be used.

4. The scoring system has been set up to give more credit in the long run for leaving a question unanswered than guessing rashly. On the other hand, whenever you can eliminate three possibilities, it is better to guess between the remaining two possibilities than to leave the question unanswered.

5. Fill in the following blank and wait for the signal to start the examination.

PRINT_____
First Name Last Name

Your teacher will fill in the following blanks:

Part	Number of Questions	Number Right	Number Not Answered
1	8	____x 4 =____	____x 1 = ____
2	8	____x 8 = ____	____x 2 =____
3	2	____x 12 = ____	____x 3 = ____
Total	18	Sub-Total _____	Sub-Total ____

Score (Sum of both sub-totals)_____

Part 1

1. How many different three-digit numbers can be made by using the three cards in the figure? Each card has to be used exactly once in a three-digit number, but the 9 cards can be turned to make a 6.

(a) 12 (b) 6 (c) 10 (d) 9 (e) 16

2. Anton, Bella and Cecil each has a cube in their hands. The three cubes are identically colored, each with faces red, green, orange, white, blue and yellow. Anton sees a vertex on his cube where faces in blue, white and yellow meet. Bella has a vertex where orange, blue and red meet. Cecil has a vertex where green, orange and white faces meet. What color is the face of the cube that is opposite to the white face?

(a) orange (b) red (c) blue (d) yellow (e) green

3. The positive integers 1, 2, …, 10 are multiplied together. How many distinct factors does the product have?

(a) 64 (b) 105 (c) 135 (d) 210 (e) 270

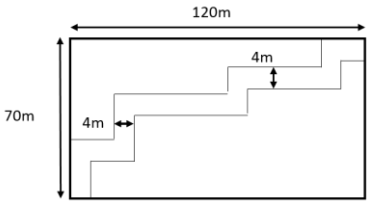
4. Jordan is swimming laps in a pool. For each lap after the first, the time it takes her to complete the lap is five seconds more than that of the previous lap. Given that she spends 10 minutes on the first six laps, how long does she spend on the next six laps, in minutes?

(a) 12 (b) 12.5 (c) 13 (d) 13.5 (e) none of these

5. Only one real value of x satisfies the equation $cx^2 + (c+3)x + 3 = 0$. What is the product of all possible values of c for which the previous statement is true?

(a) 3 (b) 12 (c) 30 (d) 1 (e) none of these

6. A walkway is constructed through a field with length 120m and width 70m. The walkway is a constant 4 m wide and turns as shown in the figure. What is the area of the walkway?



(a) 744 m^2 (b) 280 m^2 (c) 480 m^2 (d) 760 m^2 (e) 756 m^2

7. One morning Terry looks in the mirror in her bedroom, and sees that her clock fell from the wall, and is on her sofa. The figure below shows the clock as Terry sees it in the mirror. The clock's hands are identical, one points exactly at a 5-minute mark, the other is exactly half-way between two 5-minute marks. What is the time?

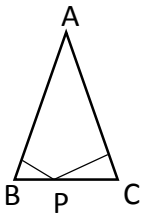
(a) 4:40 am (b) 7:00 am (c) 7:25 am (d) 10:30 am (e) 1:20 pm

8. How many statements are true on a sheet of paper that has the following statements written on it:
    1. At most one statement is true on this sheet.
    2. At most two statements are true on this sheet.
    3. At most three statements are true on this sheet.
    4. At most four statements are true on this sheet.
    5. At most five statements are true on this sheet.

(a) 4 (b) 3 (c) 2 (d) 1 (e) 0

Part 2

9. Triangle *ABC* is isosceles with *AB* = *AC* and *BC* = 65 cm. *P* is a point on *BC* such that the perpendicular distances from *P* to *AB* and *AC* are 24 cm and 36 cm, respectively. Find the area of Δ *ABC*, in cm^2 ,



(a) 2535 (b) 1254 (c) 1640 (d) 1950 (e) 2942

10. Real numbers *a*,*b* and *c* satisfy

$$\frac{-a+b+c}{a} = \frac{a-b+c}{b} = \frac{a+b-c}{c}.$$

What is the largest value of

$$p = \frac{(a+b)(b+c)(c+a)}{abc}\ ?$$

(a) 6 (b) 6.5 (c) 7 (d) 7.5 (e) 8

11. Among all the triangles that are determined by three arbitrarily chosen vertices of a cube, what is the ratio of the number of right triangles to the total number of triangles?

(a) 2:3 (b) 3:4 (c) 6:7 (d) 7:8 (e) 11:12

12. The magic square in the figure has the property that the sum of the numbers in each row, each column and the two diagonals are all the same. What is the value of M?

M		41
11	32	

(a) 10 (b) 15 (c) 17 (d) 19 (e) none of these

13. The eighteen-digit number *x*36 405 489 812 706 44*y* is divisible by 99. The value of *x* is

(a) 6 (b) 7 (c) 8 (d) 9 (e) 1

14. If 2015 can be written as the sum of an *odd* number (n) of consecutive positive integers, which of the following values can n take?

(a) 3 (b) 5 (c) 7 (d) 9 (e) 11

15. Nine parallel lines in the plane intersect *n* parallel lines, each of which is perpendicular to the nine lines. A **total** of 756 rectangles are formed by these lines. What is the value of *n*?

(a) 5 (b) 6 (c) 7 (d) 8 (e) 9

16. The product of 2016 integers is 1. Which of the following cannot be their sum

(a) 2012 (b) 2000 (c) 2006 (d) 2004 (e) 2016

Part 3

17. What is the largest positive integer that can divide both $k^2 + 7$ and $(k + 1)^2 + 7$ where *k* is a positive integer?

(a) 1 (b) 5 (c) 7 (d) 13 (e) none of these

18. The terms of a monotonically increasing sequence $\{a_i\}$, i=1,2,3, …, are positive integers. The sequence has the property that every positive integer is either a term of this sequence, or is a sum of two different terms of this sequence. Which of the following must be true?

I. $a_{29}=29$
II. $a_{100}<10{,}000$
III. $a_1=1$

(a) I only (b) I and III only (c) II only (d) II and III only (e) All