Mathematical Association of America
Wisconsin Section
Mathematics Contest Examination
December 8, 2016

1. <u>Do not open this booklet until you are directed to do so</u>.

2. This is a multiple choice test. Each multiple choice question has five possible answers, exactly one of which is correct. You are to circle the letter corresponding to the correct response on the answer sheet for as many problems as you can do in the 75 minutes allowed.

   <u>EXAMPLE:</u>
   If x is 3 and y is 4 then $2x - y$ is

   (a) –1   (b) 0   (c) 1   (d) 2   (e) none of these.

3. Use pencil or pen. <u>A sheet of paper will be provided for your scratch work</u>. Calculators may be used. Tables, books, notes, etc. may not be used.

4. <u>The scoring system has been set up to give more credit in the long run for leaving a question unanswered than guessing rashly</u>. On the other hand, whenever you can eliminate three possibilities, it is better to guess between the remaining two possibilities than to leave the question unanswered.

5. Fill in the following blank and wait for the signal to start the examination.

PRINT_____
        First Name   Last Name

Your teacher will fill in the following blanks:

| Part	Number of Questions	Number Right	Number Not Answered
1	8	____x 4 =____	____x 1 = ____
2	8	____x 8 = ____	____x 2 =____
3	2	____x 12 = ____	____x 3 = ____
Total	18	Sub-Total _____	Sub-Total ____

Score (Sum of both sub-totals)_____

Part 1

1. What is the value of the fraction $\frac{1\cdot2\cdot3+2\cdot4\cdot6+4\cdot8\cdot12}{1\cdot3\cdot5+2\cdot6\cdot10+4\cdot12\cdot20}$?

    (a) 0.2   (b) 0.4   (c) 0.6   (d) 0.8   (e) 1

2. The acute angle of the shaded rhombi in the figure is $30°$. What is the measure of the angle α in degrees?
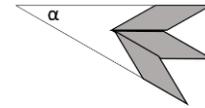
    (a) 30°   (b) 40°   (c) 45°   (d) 60°   (e) none of these

3. Susy makes up a sequence of numbers by the following rule. Each term is 7 times the sum of the digits of the previous term. So far she has the following terms: 13, 28, 70. What is the 100th term in this sequence?

   (a) 13   (b) 28   (c) 49   (d) 70   (e) 91

4. On a bus route there are a total of 8 stops. Anyone who gets on the bus travels on it until at least the next stop. On a Monday morning trip from the first to the last stop it was discovered that no two travelers got on and off at the same two stops. What is the maximum number of passengers who could have used the bus on this trip?

    (a) 24   (b) 28   (c) 32   (d) 36   (e) 40

5. 2016 is a 'super square' year, since (I) the number formed from the last two digits is a perfect square ($16=4^2$), (II) the sum of the digits is a perfect square ($2+0+1+6=3^2$), and (III) the sum of the numbers formed from the first two digits and from the last two digits is also a square ($20+16=6^2$). How many super square years are there between 1000 and 2015?

    (a) 0   (b) 1   (c) 2   (d) 3   (e) more than 3

6. The Pokemon Zig-a-Zag can only travel on zig-zagging roads. His friends made a road from his house to his practice field which is 14 miles away due East. The road starts out at a $60°$ angle from a horizontal line, and each zig forms a $60°$ angle with the zag before and after it, as shown in the figure below. Zig-a-Zag goes to the field and back each day on this road. How many miles does he travel on the road?
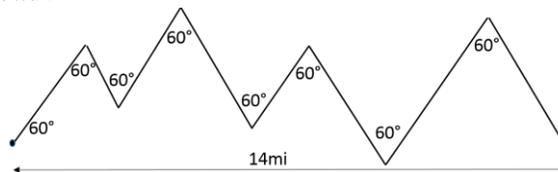
    (a) 28 mi   (b) 35 mi   (c) 42 mi   (d) 56mi   (e) not enough information

7. Two squares are arranged as shown in the figure. C is the center of the smaller square as well as a vertex of the larger one. The smaller square has side 4 in, while the larger one has side 6in. What is the area of the intersection of the two squares?

   (a) 2 sq in (b) 4 sq in (c) 6 sq in (d) 8 sq in   (e) 10 sq in
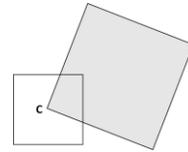
8. All these shapes have the same perimeter, except one. Which is the odd one out?
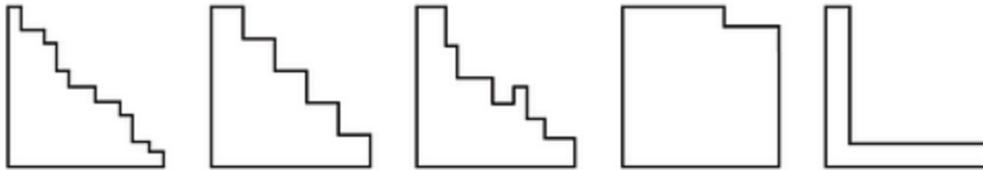
   (a)   (b)   (c)   (d)   (e)

   Part 2

9. Tanisha wrote the numbers 1 through 2016 on a piece of paper, and put the papers in a hat. How many pieces does one need to pull randomly from the hat to make sure that there are two numbers among the chosen ones whose sum is divisible by 3?

   (a) 3   (b) 4   (c) 672   (d) 673   (e) 674

10. There are three identical cubical boxes A, B and C (their widths, lengths, and heights are all the same). Box A contains 8 identical iron balls arranged in rows of two balls in all dimensions (two layers of four balls). The outside balls touch the walls of the box and the balls touch each other. Box B contains 27 identical iron balls arranged in rows of three balls in all dimensions (three layers of nine balls) in such a way that the outside balls touch the walls of the box, and the balls touch each other. Similarly, in Box C, there are 64 identical iron balls arranged in the same manner, a single layer has 16 balls, and there are four layers. Assuming that the balls are made up of the same material, which box is the heaviest?

    (a) Box A   (b) Box B   (c) Box C   (d) they weigh the same   (e) not enough information

11. How many perfect squares divide 20! evenly, where $20!=1\cdot2\cdot3\ \ldots\ \cdot20$?

    (a) 36   (b) 120   (c) 300   (d) 400   (e) 625

12. There are 15 students in a class. Which of the following statements cannot be true? (Friendship is mutual, so if Tom is friends with Joe, then Joe is friends with Tom.)
      I.   Everyone is friends with everyone else in the class.
      II.   Everybody has exactly two friends.
      III.   Everybody has exactly five friends.

    (a) only I   (b) only II (c) only III   (d) I and II   (e) II and III

13. Tom and Jerry run in opposite directions on a circular track at a constant (but different) speed, starting at diametrically opposite points. They first meet when Jerry ran 120 meters. They meet next when Tom ran 180 meters past their first meeting point. What is the length of the track in meters?

    (a) 240m   (b) 300m   (c) 360m   (d) 420m   (e) none of these

14. If you add up the 99 decimal numbers 9, 99, 999, … , 999999...99, where the last number has 99 digits, how many digits of 1 will the sum contain?

    (a) 96   (b) 97   (c) 98   (d) 99   (e) 100

15. Sasha holds 6 pieces of string in her hand as shown. Johnny ties the ends of the strings in twos both on top and in the bottom. What is the probability that when Sasha lets the strings go, they will be connected in one large loop?

    (a) 1/15   (b) 7/15   (c) 8/15   (d) 9/15   (e) 1

16. For what integer value of k does the following equation hold: $\left(\sqrt{2}-1\right)^5 = \sqrt{k+1} - \sqrt{k}$

    (a) 65   (b) 120   (c) 1023   (d) 1370   (e) 1681

Part 3

17. Let $f(x) = x^3 - x + 1$. What is the greatest common divisor of $f(2016)$ and $f\big(f(\,...\,f(2016)\,...\,)\big)$, where f is applied 2016 times in the latter expression?

    (a) 1   (b) 17   (c) 101   (d) 2017   (e) none of these

18. Karen is commuting by train to the city to take classes. Her father picks her up at the same time each day at the train station, and gives her a ride home. Karen's father arrives at the station exactly the same time as her train. One day Karen's last class got canceled, and she took an earlier train. When she arrived at the station she started to walk along the route that her father usually took. She walked for 30 minutes when they met, and then they rode home together, arriving 20 minutes earlier than usual. How much earlier did Karen's train arrive than her usual one?

    (a) 20 min   (b) 35 min (c) 40 min   (d) 45 min   (e) 50 min