Mathematical Association of America
Wisconsin Section
Mathematics Contest Examination
December 7, 2017

1. Do not open this booklet until you are directed to do so.

2. This is a multiple choice test. Each multiple choice question has five possible answers, exactly one of which is correct. You are to circle the letter corresponding to the correct response on the answer sheet for as many problems as you can do in the 75 minutes allowed.

   EXAMPLE:
   If x is 3 and y is 4 then $2x - y$ is

   (a) –1 (b) 0 (c) 1 (d) 2 (e) none of these.

3. Use pencil or pen. A sheet of paper will be provided for your scratch work. Calculators may be used. Tables, books, notes, etc. may not be used.

4. The scoring system has been set up to give more credit in the long run for leaving a question unanswered than guessing rashly. On the other hand, whenever you can eliminate three possibilities, it is better to guess between the remaining two possibilities than to leave the question unanswered.

5. Fill in the following blank and wait for the signal to start the examination.

PRINT_____
First Name Last Name

Your teacher will fill in the following blanks:

Part	Number of Questions	Number Right	Number Not Answered
1	8	____x 4 =____	____x 1 = ____
2	8	____x 8 = ____	____x 2 =____
3	2	____x 12 = ____	____x 3 = ____
Total	18	Sub-Total _____	Sub-Total ____

Score (Sum of both sub-totals)_____

Part 1

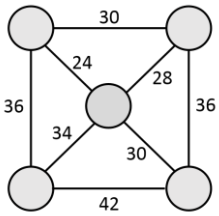
1. Which of the following fractions has the smallest value?

   (a) 9/7 (b) 88/66 (c) 777/555 (d) 6666/4444 (e) 55555/33333

2. The operation ⌂ is defined by $a \text{ ⌂ } b = a^3b - b^3a$. The value of -2 ⌂ 3 is

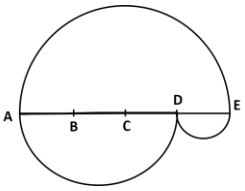
   (a) -6 (b) 6 (c) 30 (d) 42 (e) none of these

3. What is the sum of the five numbers that you could put in the shaded circles so that the numbers given on each line segment would be the sum of the numbers written in the shaded circles at the two endpoints of the segment?



   (a) 28 (b) 49 (c) 70 (d) 83 (e) 91

4. In the figure, AE is divided into four equal parts. Three semicircles are constructed taking AE, AD, and DE as diameters, and creating two paths from A to E, as shown. What is the ratio of the length of the upper path to the length of the lower path?

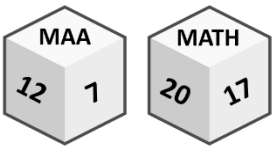


   (a) 1:2 (b) 2:3 (c) 2:1 (d) 3:2 (e) 1:1

5. Angle β is 25% less than angle γ and 50% greater than angle α. Which of the following is true about angle γ?

   (a) It is 25 % greater than α. (b) It is 50% greater than α. (c) It is 75% greater than α. (d) It is 100% greater than α. (e) It is 125% greater than α.

6. The figure shows two different views of the same die. The numbers 7 and 17 are on opposite sides of the die. What is written on the opposite of the side with MAA on it?



   (a) 7 (b) MATH (c) 12 (d) 17 (e) 20

7. Let [x] be the greatest integer less than or equal to x. For example [3.7] = 3 and [-3.7] = -4. What is the value of x such that x·[x] = 14?

   (a) 7/2 (b) -7/2 (c) 4 (d) -4 (e) -9/2

8. The real number $\sqrt{19-8\sqrt{3}}$ can be expressed in the form of $a+b\sqrt{3}$ where a and b are integers and a is positive. What is the value of $a + b$?

   (a) 3 (b) 4 (c) 5 (d) 6 (e) 7

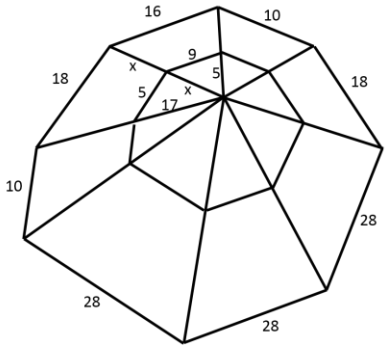
Part 2

9. N has 10 positive divisors, 2N has 15 positive divisors, 3N has 20 positive divisors. How many positive divisors does 4N have?

   (a) 15 (b) 20 (c) 24 (d) 28 (e) 32

10. A spider spins a web and some of the strings have lengths as shown in the figure. If x is an integer, then the value of x is

    (a) 11 (b) 13 (c) 15 (d) 17 (e) 19



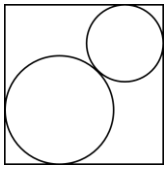
11. Alan and Bertrand run on a circular track. Alan runs counterclockwise and completes a lap every 80 seconds, while Bertrand runs clockwise and completes a lap every 90 seconds. Both start from the start line at the same time. At some random time between ten minutes and eleven minutes after they begin to run, a photographer standing inside the track takes a picture that shows one-fourth of the track, centered on the starting line. What is the probability that both Alan and Bertrand are in the picture?

    (a) 1/4 (b) 1/8 (c) 3/8 (d) 1/2 (e) 3/16

12. In Dragonland every dragon has at least 3 heads. Dragons that have an odd number of heads always tell the truth, dragons that have an even number of heads always lie. Four dragons were having a barbecue, when they were asked how many heads the four of them have in total. They answered 14, 15, 16 and 20. Which of the following is a possible total number of heads for the truth-telling dragons in the company?

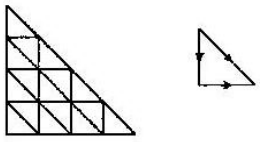
    (a) 3 (b) 6 (c) 5 (d) 7 (e) 9

13. Two circles have their centers on one of the diagonals of a square. They touch each other and the sides of the square, as shown. The square has a side length of 1 cm. What is the sum of the lengths of the two radii of the circles, in centimeters?



(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}-1$ (d) $2-\sqrt{2}$ (e) none of these

14. A map of a neighborhood is shown on the figure. The neighborhood has the form of a right triangle. All streets are shown on the map. They divide the neighborhood into blocks that are also right triangles. Alex wants to drive from the top endpoint of the neighborhood to the rightmost endpoint of the neighborhood. The traffic regulations only allow going down (vertically), right (horizontally), or down by a "hypotenuse" of a block. From how many different routes can Alex choose?



(a) 16 (b) 27 (c) 64 (d) 90 (e) 111

15. Two sticks have length *a* and *b* with *a* > *b*. You break the longer stick at a random point. What is the probability that the resulting three sticks form a triangle?

(a) b/a (b) 1/2 (c) b/2a (d) 2b/a (e) none of these

16. What is the value of the sum $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots\frac{1}{\sqrt{99}+\sqrt{100}}$?

(a) 9 (b) 10 (c) 99 (d) $\sqrt{99}$ (e) $\sqrt{99}-1$

Part 3

17. Let $f(x)=x^3-x+1$. What is the greatest common divisor of $f(2016)$ and $f\big(f(\ldots f(2016)\ldots)\big)$, where f is applied 2016 times in the latter expression?

(a) 1 (b) 17 (c) 101 (d) 2017 (e) none of these

18. Let *S(x)* denote the sum of the digits of the positive integer *x*. How many solutions does the equation $x+S(x)+S\big(S(x)\big)=2017$ have?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 2017