Mathematical Association of America
Wisconsin Section
Mathematics Contest Examination
December 6, 2018

1. Do not open this booklet until you are directed to do so.

2. This is a multiple choice test. Each multiple choice question has five possible answers, exactly one of which is correct. You are to circle the letter corresponding to the correct response on the answer sheet for as many problems as you can do in the 75 minutes allowed.

   EXAMPLE:
   If x is 3 and y is 4 then $2x - y$ is
   (a) –1 (b) 0 (c) 1 (d) 2 (e) none of these.

3. Use pencil or pen. A sheet of paper will be provided for your scratch work. Calculators may be used. Tables, books, notes, etc. may not be used.

4. The scoring system has been set up to give more credit in the long run for leaving a question unanswered than guessing rashly. On the other hand, whenever you can eliminate three possibilities, it is better to guess between the remaining two possibilities than to leave the question unanswered.

5. Fill in the following blank and wait for the signal to start the examination.

PRINT_____
First Name Last Name

Your teacher will fill in the following blanks:

Part	Number of Questions	Number Right	Number Not Answered
1	8	____x 4 =____	____x 1 = ____
2	8	____x 8 = ____	____x 2 =____
3	2	____x 12 = ____	____x 3 = ____
Total	18	Sub-Total _____	Sub-Total ____

Score (Sum of both sub-totals)_____

1. How many integers are there between 1 and 2018 whose digits add up to 3 ?

   a) 13
   b) 14
   c) 16
   d) 17
   e) 18

2. If $x + y = 4$ and $x^3 + y^3 = 28$, find xy.
   a) -2
   b) 3
   c) $9/2$
   d) 7
   e) 9

3. Given $\frac{3a - b}{a^2 - 4} = 0$, which of the following must be true ?

   i. $a \neq \pm 2$
   ii. $b = 3a$
   iii. $b \neq \pm 6$
   iv. $a < b$

   a) Only I
   b) I and II
   c) I and IV
   d) III and IV
   e) I, II and III

4. A family with 5 people consists of mom, dad, grandpa, and two kids: a girl and a boy. Grandpa is (ab) years old, dad (ba), mom (bb), girl (a), and the boy (b). (Note a, b are digits). In this family, the average age of the males is 38,and the average age of the females is 25. What is the average of kids' age?

   a) 3
   b) 4
   c) 5
   d) 6
   e) 7

5. How many real numbers satisfy $x(-x) = x + \frac{1}{x}$?

   a) 0
   b) 1
   c) 2
   d) 3
   e) 4

6. If $\left(\frac{1}{4}\right)^a = \frac{1}{5}$, $\left(\frac{1}{5}\right)^b = \frac{1}{6}$, $\left(\frac{1}{6}\right)^c = \frac{1}{7}$, and $\left(\frac{1}{7}\right)^d = \frac{1}{8}$. Find $abcd$.

   a) $\frac{3}{2}$
   b) $\frac{4}{3}$
   c) $\frac{5}{3}$
   d) $\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)\left(\frac{1}{6}\right)\left(\frac{1}{7}\right)$
   e) $\left(\frac{1}{5}\right)\left(\frac{1}{6}\right)\left(\frac{1}{7}\right)\left(\frac{1}{8}\right)$

7. If $f(x) = ax^3 + bx^2 + cx + d$, $f(1) = -2$, $f(-1) = -12$, $f(2) = 9$, and $f(0) = -3$, what is $f(-2)$?

   a) -50
   b) -47
   c) -21
   d) -9
   e) 16

8. How many positive cubes divide $(3!)(5!)(7!)$?

   a) 3
   b) 4
   c) 5
   d) 6
   e) 7

9. Two people are running a 100 meter race. They both run at a constant speed and runner A wins by $10\ m$. After resting for a while, they race again with runner A starting $10\ m$ behind the original starting line. If they run the same speeds as before, who wins and by how much ?

   a) A by $2\ m$
   b) A by $1\ m$
   c) They tie
   d) B by $1\ m$
   e) B by $2\ m$

10. What is the highest power of 28 which divides $(2018!)$?

   a) 276
   b) 278
   c) 322
   d) 328
   e) 334

11. What is the units digit of $(19)^{99} + (99)^{99}$?

   a) 0
   b) 2
   c) 4
   d) 6
   e) 8

12. If you have an unlimited supply of 5 cent and 11 cent postage stamps, what is the largest postage you cannot make up exactly ?

   a) 23 cents
   b) 28 cents
   c) 39 cents
   d) 47 cents
   e) 51 cents

13. How many positive integers less than 100 are not prime and are not divisible by $2, 3,$ or 5 ?

a) 3
b) 4
c) 5
d) 6
e) 7

14. Determine $12 - 22 + 32 - 42 + 52 \ldots\ldots - 20182 + 20192$.

a) 12
b) 10102
c) 30282
d) 2039190
e) 20395938

15. Find the value $\frac{e^x}{e^y}$ given that $\frac{\frac{1}{e^x} + \frac{1}{e^y}}{\frac{1}{e^x} - \frac{1}{e^y}} = 2018$.

a) $\frac{2017}{2019}$
b) $\frac{2019}{2017}$
c) $\ln(2019) - \ln(2017)$
d) $\ln(2017) - \ln(2019)$
e) $\frac{1}{2017} + \frac{1}{2019}$

16. Determine the approximate perimeter of a regular pentagon given that the distance from the center to vertex is 2018 inches.

a) $11{,}856$ *in.*
b) $11{,}858$ *in.*
c) $11{,}860$ *in.*
d) $11{,}862$ *in.*
e) $11{,}864$ *in.*

17. If $f_1(x) = \dfrac{x}{1-x}$ and $f_{n+1} = f_1 \circ f_n$ for $n = 1, 2, 3, 4, \ldots$. Evaluate $f_n(n)$.

a) $\dfrac{n^n}{(1-n)^n}$

b) $\dfrac{1+n^2}{1-n^2}$

c) $\dfrac{n}{1-n^2}$

d) $\dfrac{n}{1-n}$

e) $\dfrac{1}{1-n}$

18. A $12-$ sided convex polygon inscribed in a circle has, in some order, six sides of length $\sqrt{2}$ and six sides of length $\sqrt{24}$. Find the area of the circle.

a) 12π

b) 22π

c) 26π

d) 38π

e) 48π